number of dwellings of the oldest age represented in the group.

In any group of objects which last for varying lengths of time but in which the number of objects is kept constant by replacing discarded ones by new ones the following principles apply:

1. The number of old objects discarded each year is, on the average, equal to the number of new ones introduced.

2. The average number of objects in the second year of their life at a given time is equal to the average number of those in their first year that will live to enter their second year. The average number in their third year is on the average equal to the number of those in their first year that will live to enter their third year, and so on. In general, the number of objects in their nth year is equal to the number of those in their first year that will live to enter their nth year.

3. Hence N_2, N_3, etc., which represent the number of objects now in their second, third, etc., years of life, may also be taken to represent the number of the objects now in their first year that will ultimately reach their second, third, etc., years of life.

4. If now we add together N_1, N_2, N_3, etc., this is equivalent to counting each object now in its first year as many times as it will live years. Hence the sum of N_1, N_2, N_3, etc., which represents the total number of objects of all ages, also represents the sum of the ages that will be attained by all the objects now in their first year.

5. Therefore, if we divide the total number of objects of all ages in the group by the average number in their first year the quotient will be the average length of life that those now in their first year will live. But since the average number of objects in their first year is the same from year to year, this average is a general one and applies to the whole population. We may thus express the average length of life of any constant population by means of the following formula:

$$L = \frac{N_1 + N_2 + N_3 + \cdots + N_n}{N_1}. \qquad (A)$$

This formula may be expressed more simply by writing for the numerator simply the total population instead of the sum of individuals of different ages. We thus have

$$L = \frac{P}{N_1}. \qquad (B)$$

In this formula L equals the average length of life, P the total population, and N_1 the average number in their first year of life at a given time.

In applying either of the above formulæ to cases like those of farm houses and most kinds of farm implements the fact that very few such objects are discarded until they are at least four or five years old makes N_1, N_2, N_3, N_4 and N_5 approximately equal. That is, the number of objects one year old is about the same as the number two years old, or three years, etc., up to about five years, and sometimes even longer. In making a study of such objects with a view to determining the average length of their life it is usually possible to get quite accurately the number of objects in the group in each year of life up to five or six years of age, and where these numbers are about the same for each year their averages will represent quite accurately the average number of new objects introduced in a year, which is the same as the average number of old ones discarded. Hence, in populations where the number of objects in each of the earlier years of life is approximately the same, the average length of life in the population may be obtained by dividing the total number of objects by the average number in each of the early years of life.

POPULATIONS THAT ARE DECREASING OR INCREASING

The principles stated above do not apply in a population that does not remain constant from year to year. It is not difficult, however, to work out a formula based on formula (A) above that does apply to such populations. This may be done as follows:

Suppose the rate of increase in population is 1 per cent. a year. Then if P represents the population in any one year, $1.01P$ will repre-

sent the population the next year. Likewise, if B represents the number of births in any year, then $1.01B$ will represent the number the next year. In general, if B represents the number of births in any year and r the annual rate of increase in population, then $(1+r)B$ will represent the number of births the first year thereafter, $(1+r)^2B$ the number of births the second year thereafter, and $(1+r)^nB$ the number of births the nth year thereafter.

Returning now to formula (A), where N_1 represents the number of individuals in the first year of life, N_2 the number in their second year, and so on, we have already seen that in a constant population these numbers bear such relation to each other that N_2 represents the number of the present N_1's that will live to enter their second year. But in an increasing population this is not the case, for the number of individuals born in the year in which the present N_2's were born was smaller than the number born in the year in which the present N_1's were born—that is, the number born last year is smaller than the number born this year. Hence, in an increasing population N_2 is smaller than the number of N_1's that will live to enter their second year. But if we increase N_2 in proportion as the number born this year is greater than the number born last year, this increased value of N_2 will represent the number of the present N_1's that will live to enter their second year.

If we let B stand for the number born in the year in which the present N_2's were born, then $(1+r)B$ will represent the number born the year the present N_1's were born, which of course is just one year later. The increased value of N_2, for which we are seeking, may now be found from the proportion

$$B : (1+r)B :: N_2 : X,$$

from which

$$X = (1+r)N_2.$$

In similar manner it can be shown that if we substitute for N_3 the expression $(1+r)^2N_3$ this new value will represent the number of present N_1's that will live to enter their third year, and so on for all of the various N's in the numerator of formula (A). This gives us

$$L = \frac{N_1 + (1+r)N_2 + (1+r)^2N_3 + \cdots + (1+r)^{n-1}N_n}{N_1}. \quad (C)$$

In this new formula the terms of the numerator represent, respectively, the number of the present N_1's that will live to enter the various years of life indicated by the subscripts after the N's. Hence the sum of the terms of the numerator is equal to the sum of the ages the present N_1's will reach at death, and the value of the whole fraction becomes the average length of life of the population.

To use formula (C), which applies to populations that are increasing or decreasing at a constant rate, r, we must know the number of individuals in each of the various years of life at the present time and the annual rate of increase or decrease in population. Such data are usually not available except in the cases of human beings in restricted areas where births and deaths are accurately recorded. In some cases, however, it may be possible to obtain data of this kind concerning a class of articles of farm equipment. When this is possible, the average length of life may be calculated where the number of objects is increasing or decreasing at a constant rate per year.

It will be noticed that when r is equal to zero, which it is in a constant population, formula (C) becomes identical with formula (A).

Formula (C) applies only to populations in which the *rate* of increase or decrease is the same from year to year. It is possible to develop another formula for the average length of life which is independent of the rate of increase and which therefore applies to any kind of population, no matter what the rate of increase or decrease, and whether this rate is the same from year to year or not.

Let B_3 represent the number of individuals born the year the present N_3 individuals were born, and B_1 the number born the present year. Then the proportion $B_3 : B_1 :: N_3 : X$, in which X is equal to $(B_1/B_3)N_3$, gives a value which if used instead of N_3 makes the third term of the numerator of formula (A) represent the

number of the present N_1's who will live to enter their third year. The other terms of the numerator of formula (A) may be modified in similar manner, giving the formula

$$L=\frac{N_1+\frac{B_1}{B_2}N_2+\frac{B_1}{B_2}+N_3\cdots+\frac{B_1}{B_n}N_n}{N_1}, \quad (D)$$

which is applicable to all populations for which we have the following data: the number of individuals born each year since and including the year in which the oldest individuals now living were born, and the number of people of various ages now living.

While formula (D) has very wide applicability, its usefulness is greatly limited by the fact that it requires so large an amount of data which is usually difficult to obtain.

Before applying any of these formulæ it is necessary to eliminate the effect of immigration and emigration. This means that only those individuals should be considered whose whole life is to be spent as a part of the population under consideration. In using any of the methods here presented in determining the average rate of depreciation of, say, a farm implement of a given kind, only those implements are to be counted that were bought new (not second hand) and which will presumably be replaced when destroyed or worn out by new ones.

W. J. SPILLMAN

WASHINGTON, D. C.

QUARTER CENTENNIAL OF THE IOWA ACADEMY OF SCIENCE

ON Friday and Saturday, April 26–27, the Iowa Academy of Science celebrated the twenty-fifth anniversary of its organization. The sessions were held in the Art Gallery of the State Historical Building in Des Moines, beginning at 1:30 Friday afternoon. The president's address was given by Professor Louis Begeman, of the State Teachers College, on "The Mission and Spirit of the Pure Scientist." After the president's address the reading of the usual scientific papers as presented before the academy was the order of the afternoon. As forty-six titles were presented, it was necessary that the time allotted to each paper be very brief.

The anniversary banquet was held Friday evening at the Chamberlain Hotel, with an attendance of seventy. At the close of the banquet short addresses of congratulation were given by representatives of neighboring scientific societies. The Nebraska Academy of Science was represented by Professor Addison E. Sheldon, the Illinois Academy of Science by Professor Henry B. Ward, the Davenport Academy by Professor C. C. Nutting, the St. Louis Academy by Professor L. H. Pammel, the American Microscopical Society by Professors H. E. Summers and L. S. Ross and the Ohio Academy by Professor Herbert Osborn. An address on the "Charter Members" was given by Professor L. H. Pammel and the anniversary address by Professor Herbert Osborn, of the State University of Ohio, the first president of the Iowa Academy.

In accordance with the purpose of the anniversary meeting, the session Saturday forenoon was devoted to addresses on "The Development of the Sciences in Iowa during the Past Twenty-five Years":

Botany—Professor Thomas H. Macbride.
Chemistry—Professor W. S. Hendrixson.
Geology—Professor M. F. Arey.
Physics—Professor Frank F. Almy.
Zoology—Professor C. C. Nutting.

These papers gave valuable reviews of the status of the sciences in the colleges at the time of the organization of the academy, and historical sketches indicating marked advance in all scientific lines during the quarter century.

At the business meeting, over eighty applications for membership were presented. The meeting was in every way fitting to celebrate the end of a quarter century of earnest and effective work done by the academy.

The 1913 meeting will be held at the Iowa State College, Ames.

TITLES OF PAPERS PRESENTED

(Abstracts furnished by authors)

Ferns and Liverworts of Grinnell and Vicinity: H. S. Conard.

Secotium warnei, a Stalked Puffball: H. S. Conard.

Simblum rubescens in Iowa: H. S. Conard.

Aroid Notes: James Ellis Gow.

In studying the morphology of some twenty species of Aroids, mostly tropical, the writer found that there is great confusion in the nomenclature of the species. A reference to the original sources has made it possible to give a correct account of the taxonomy of all but one species, and the results are here presented.